Particle creation in an isotropic universe in the framework of the extended phase space approach

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The main idea of our research:

The application of the extended phase space approach to quantization of gravity to particle creation in the Early Universe.

Quantum field theory on a curved background:

- The gravitational field is classical.
- Matter fields are quantized in the canonical framework.
- The matter Hamiltonian is diagonalized at each instant of time by means of a time dependent Bogoliubov transformation, so we have an "instant" Hamiltonian operator and an "instant" Fock basis.

The features of the extended phase space approach to quantization of gravity:

- The path integral is used to derive a Schrödinger equation for the wave function that depends on geometry of the Universe and matter fields. So, gravity is quantized as well as matter.

- We consider the path integral with an effective action including gauge fixing and ghost terms.

- In gauge theories, the path integral is usually considered under asymptotic boundary conditions that ensure its gauge invariance. Asymptotic boundary conditions imply that asymptotic "in" and "out" states do exist, that takes place in ordinary quantum theory.

- In the theory of gravity, the path integral gives a transition amplitude between two spacelike hypersurfaces. Physical states on these hypersurfaces do not have to be asymptotic. The asymptotic boundary conditions are justified only in asymptotically flat spacetimes.

- Since we cannot impose the asymptotic boundary conditions in the path integral, we cannot prove its gauge invariance.

- The Schrödinger equation, that we obtained from the path integral, is gauge dependent. In its turn, the Wheeler-DeWitt equation loses its sense.

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We consider scalar and spinor fields on the background of a closed isotropic universe.

$$ds^2 = N^2(t)\,dt^2 - a^2(t)\left[d\chi^2 + \sin^2\chi\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right]$$

The gravitational action is $$S_{grav} = \frac{1}{2}\int dt\left(-\frac{a\dot{a}^2}{N} + Na\right)$$

The gauge-fixing and ghost sectors are given by: $$S_{gf} = \int dt\left[\pi\left(\dot{N} - \frac{df}{da}\dot{a}\right)\right]; \quad S_{ghost} = \int dt\,\dot{\bar{\theta}}\,N\theta.$$

The matter action:

$$S_{mat} = \frac{1}{4\pi^2}\int d^4x\sqrt{-g}\left[g^{\mu\nu}\partial_\mu\phi^*\partial_\nu\phi + \frac{1}{6}(1+\xi)R\phi^*\phi + i\bar{\psi}e^\mu_m\gamma^m D_\mu\psi\right]$$

The scalar and spinor fields are introduced to regularize vacuum energy divergences using supersymmetric multiplets.

$\xi = 0$ corresponds to a conformal scalar field. In this case, the Schrödinger equation can be solved exactly.

At the next stage, we obtain the Schrödinger equation for a non-conformal scalar field considering ξ as a small parameter, and the terms proportional to ξ as a perturbation.

$$S_{mat}=\frac{1}{4\pi^2}\int d^4x\sqrt{-g}\left[g^{\mu\nu}\partial_\mu\phi^*\partial_\nu\phi+\frac{1}{6}R\phi^*\phi+i\overline{\psi}e^\mu_m\gamma^m D_\mu\psi\right]$$

We redefine the scalar and spinor fields as $$\phi\to\sqrt{2}\pi\frac{\phi}{a},\quad \psi\to\sqrt{2}\pi\frac{\psi}{a^{2/3}}$$

Then we decompose these fields into eigenfunctions of the Laplace–Beltrami operator in a positive curvature space Φ_{nlm} and spinor spherical harmonics Ψ_{njlm}.

For the scalar field we have:

$$\phi(\mathbf{x},t)=\sum_{\mathbf{k}}\frac{1}{\sqrt{2\lambda_{\mathbf{k}}}}\left[b_{\mathbf{k}}(t)\Phi_{\mathbf{k}}(\mathbf{x})+c^*_{\mathbf{k}}(t)\Phi^*_{\mathbf{k}}(\mathbf{x})\right]$$

k={n, l, m} is the compact notation.

It follows from Hamilton equations that $$\dot{\phi}(\mathbf{x},t)=i\frac{N}{a}\sum_{\mathbf{k}}\sqrt{\frac{\lambda_{\mathbf{k}}}{2}}\left[b_{\mathbf{k}}(t)\Phi_{\mathbf{k}}(\mathbf{x})-c^*_{\mathbf{k}}(t)\Phi^*_{\mathbf{k}}(\mathbf{x})\right]$$

Performing integration in the matter action over space coordinates, we would come to the action expressed though "holomorphic variables“ $b_{\mathbf{k}}(t)$, $c_{\mathbf{k}}(t)$ and their complex conjugates.

In the path integral, we use a mixed representation: a coordinate representation for gravitational variables and ghosts and the so-called holomorphic representation for scalar and spinor fields.

The coordinate representation:

R. P. Feynman, Space-time approach to non-relativistic quantum mechanics, *Rev. Mod. Phys*. **20** (1948), P. 367.

$$\psi\left(x(t+\varepsilon),t+\varepsilon\right)=\int\exp\left[\frac{i}{\hbar}S\left(x(t+\varepsilon),x(t)\right)\right]\psi\left(x(t),t\right)\frac{dx(t)}{A}$$

$$\left\langle x(t+\varepsilon)\middle|\Psi(t+\varepsilon)\right\rangle=\int\left\langle x(t+\varepsilon)\middle|\hat{U}(t,t+\varepsilon)\middle|x(t)\right\rangle\left\langle x(t)\middle|\Psi(t)\right\rangle\frac{dx(t)}{A}$$

To derive the Schrödinger equation, one should approximate the action on a small time interval $[t, t+\varepsilon]$. It implies the approximation of velocities.

$$x(t)=x(t')-\varepsilon\dot{x}(t')+\frac{\varepsilon^2}{2}\ddot{x}(t')-\frac{\varepsilon^3}{3!}\dddot{x}(t')+\ldots\quad t'=t+\varepsilon$$

$$\dot{x}(t)=\frac{z}{\varepsilon}+\frac{\varepsilon}{2}\ddot{x}(t')-\frac{\varepsilon^2}{3!}\dddot{x}(t')+\ldots\quad z=x(t')-x(t)$$

The coordinate representation:

Then, one should expand the action in power of z, expand the wave function as a Taylor series and perform the so-called Gaussian quadratures.

In the example above a measure in the path integral is trivial, that is not always valid. In the zero order according to ε, one would get an equation determining the measure, while in the first order, one would obtain the Schrödinger equation.

$$i\hbar\frac{\partial\psi}{\partial t}=-\frac{\hbar^2}{2}\frac{1}{\sqrt{g}}\frac{\partial}{\partial x^i}\left(\sqrt{g}\,g^{ij}\frac{\partial\psi}{\partial x^j}\right)+\frac{\hbar^2}{6}R\psi$$

This equation was obtained by Cheng for Lagrangians quadratic in velocities.

$$\mathcal{L}\left(x,\dot{x}\right)=\frac{1}{2}g_{ij}\left(x\right)\dot{x}^i\dot{x}^j$$

Here g_{ij} is a metric of configurational space, g is its determinant, R is the curvature of configurational space. The quantum correction proportional to $\hbar^2$ and R is a feature of the path integral approach.

K. S. Cheng, Quantization of a general dynamical system by Feynman's path integration formulation, *J. Math. Phys*. **13** (1972), P. 1723.

The holomorphic representation:

It corresponds to the second quantization representation in the canonical framework. In the case of one-dimensional harmonic oscillator, the starting point for the derivation of the Schrödinger equation is

$$\psi\left(b\left(t+\varepsilon\right),t+\varepsilon\right)=\int U\left(b^{*},\beta,t,t+\varepsilon\right)\psi\left(\beta^{*}\left(t\right),t\right)\exp\left(-\beta^{*}\beta\right)\frac{d\beta^{*}d\beta}{2\pi i}$$

$U(b^*,\beta,t.t+\epsilon)$ is a kernel of the evolution operator, which is given by

$$U\left(b^{*},\beta,t,t+\varepsilon\right)=\exp\left[b^{*}\beta-i\varepsilon H\left(b^{*},\beta\right)\right]$$

The Hamiltonian $H\left(b^{*},\beta\right)=\omega b^{*}\beta$

Again, after expanding the wave function $\psi(\beta^*(t),t)$ as a Taylor series and integration, we would come to the Schrödinger equation for the harmonic oscillator

$$i\hbar\frac{\partial\psi}{\partial t}=\omega b^{*}\frac{\partial\psi}{\partial b^{*}}\quad\Rightarrow\quad i\hbar\frac{\partial\psi}{\partial t}=\omega\hat{b}^{\dagger}\hat{b}\psi$$

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The case $\xi = 0$.

$$\Psi\left(N',a',b_{\mathbf{k}}^{*\prime},c_{\mathbf{k}}^{*\prime},t+\varepsilon\right)=\int\exp\left[iS_{grav+gf+ghost}\left(N',N,a',a,\bar{\theta}',\bar{\theta},\theta',\theta\right)\right]\exp\left[iS_{scal}\left(b_{\mathbf{k}}',b_{\mathbf{k}},c_{\mathbf{k}}',c_{\mathbf{k}}\right)\right]\times$$

$$\times\Psi\left(N,a,\bar{\theta},\theta,b_{\mathbf{k}}^{*},c_{\mathbf{k}}^{*},t\right)\prod_{\mathbf{k}}\exp\left(-b_{\mathbf{k}}^{*}b_{\mathbf{k}}-c_{\mathbf{k}}^{*}c_{\mathbf{k}}\right)\frac{db_{\mathbf{k}}^{*}db_{\mathbf{k}}}{2\pi i}\frac{dc_{\mathbf{k}}^{*}dc_{\mathbf{k}}}{2\pi i}d\pi dN da d\bar{\theta}d\theta$$

$$\Psi\left(N',a',b_{\mathbf{k}}^{*\prime},c_{\mathbf{k}}^{*\prime},t+\varepsilon\right)=\int U_{grav+gf+ghost}\left(N',N,a',a,\bar{\theta}',\bar{\theta},\theta',\theta,t,t+\varepsilon\right)\Psi\left(N,a,\bar{\theta},\theta,t\right)\times$$

$$\times\prod_{\mathbf{k}}U_{scal}\left(b_{\mathbf{k}}^{*\prime},b_{\mathbf{k}},c_{\mathbf{k}}^{*\prime},c_{\mathbf{k}},t,t+\varepsilon\right)\Psi\left(b_{\mathbf{k}}^{*},c_{\mathbf{k}}^{*},t\right)\exp\left(-b_{\mathbf{k}}^{*}b_{\mathbf{k}}-c_{\mathbf{k}}^{*}c_{\mathbf{k}}\right)\frac{db_{\mathbf{k}}^{*}db_{\mathbf{k}}}{2\pi i}\frac{dc_{\mathbf{k}}^{*}dc_{\mathbf{k}}}{2\pi i}d\pi dN da d\bar{\theta}d\theta$$

$$U_{scal}\left(b_{\mathbf{k}}^{*\prime},b_{\mathbf{k}},c_{\mathbf{k}}^{*\prime},c_{\mathbf{k}},t,t+\varepsilon\right)=\exp\left[b_{\mathbf{k}}^{*\prime}b_{\mathbf{k}}+c_{\mathbf{k}}^{*\prime}c_{\mathbf{k}}-i\varepsilon\frac{N}{a}\lambda_{\mathbf{k}}\left(b_{\mathbf{k}}^{*\prime}b_{\mathbf{k}}+c_{\mathbf{k}}^{*\prime}c_{\mathbf{k}}\right)\right]$$

The change of variables: $n=N'-N$; $z=a'-a$; $\bar{\eta}=\bar{\theta}'-\bar{\theta}$; $\eta=\theta'-\theta$

The part of the integral over holomorphic variables does not depend on these new variables, so we can treat the integral over holomorphic variables and the integral over coordinate variables separately.

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We can consider our model without including ghosts, then we come to the equation that completely coincides with the one obtained by means of canonical quantization. For the system of gravitational, scalar and spinor fields the equation has the form:

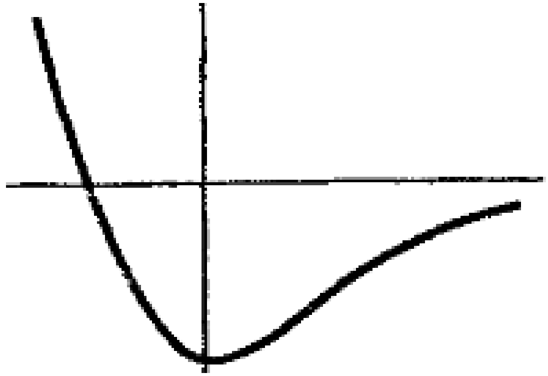
$$i\hbar\frac{\partial\Psi}{\partial t}=\frac{1}{2}\sqrt{\frac{N}{a}}\frac{\partial}{\partial a}\left(\sqrt{\frac{N}{a}}\frac{\partial\Psi}{\partial a}\right)-\frac{1}{2}Na\Psi+\frac{N}{a}\left[\sum_{\mathbf{k}}\lambda_{\mathbf{k}}\left(b_{\mathbf{k}}^{*}\frac{\partial\Psi}{\partial b_{\mathbf{k}}^{*}}+c_{\mathbf{k}}^{*}\frac{\partial\Psi}{\partial c_{\mathbf{k}}^{*}}\right)-\sum_{\mathbf{p}}\lambda_{\mathbf{p}}\left(\beta_{\mathbf{p}}^{*}\frac{\partial\Psi}{\partial\beta_{\mathbf{p}}^{*}}+\gamma_{\mathbf{p}}^{*}\frac{\partial\Psi}{\partial\gamma_{\mathbf{p}}^{*}}\right)\right]$$

$$E\Psi=\frac{1}{2}\sqrt{\frac{N}{a}}\frac{\partial}{\partial a}\left(\sqrt{\frac{N}{a}}\frac{\partial\Psi}{\partial a}\right)-\frac{1}{2}Na\Psi+\frac{N}{a}\varepsilon_N\Psi$$

$$\Psi=\psi(a)\,\chi\left(b_{\mathbf{k}}^{*},c_{\mathbf{k}}^{*},\beta_{\mathbf{p}}^{*},\gamma_{\mathbf{p}}^{*}\right)$$

$$\varepsilon_N=\sum_{\mathbf{k}}\lambda_{\mathbf{k}}\left(n_{1\mathbf{k}}+n_{2\mathbf{k}}+1\right)-\sum_{\mathbf{p}}\lambda_{\mathbf{p}}\left(n_{1\mathbf{p}}+n_{2\mathbf{p}}-1\right)$$

$n_{1\mathbf{k}}, n_{2\mathbf{k}}, n_{1\mathbf{p}}, n_{2\mathbf{p}}$ are bosonic and fermionic occupation numbers, respectively.



Choosing the gauge $N = a^3$ and reparametrizing the scalar factor $a = e^q$ we get

$$-\frac{d^2\psi}{dq^2}+\left(e^{4q}-2e^{2q}\varepsilon_N+2E\right)\psi=0$$

$$-\frac{d^2\psi}{dq^2}+\left(e^{4q}-2e^{2q}\varepsilon_N+2E\right)\psi=0$$

The exact solution to this equation is $\psi_n(a)=A_n e^{-\frac{a^2}{2}}a^{\sqrt{2E}}F\left(-n,\sqrt{2E}+1,a^2\right)$ $\qquad E_n=\frac{1}{2}\left(\varepsilon_N-2n-1\right)^2$

A_n is a normalization constants, F is the hypergeometric function. The quantum number n = 0, 1, 2, . . . , labels the energy eigenstates.

When ghosts are included, we alter the configurational space. It results in changing the measure in the path integral (it is equal to the square root of determinant of the configurational space metric). Also, the quantum correction appears which is proportional to the Planck constant squared and the curvature of the configurational space.

In this case, we obtain a general solution depending on ghosts. After exclusion of ghosts, we have

$$-\frac{d^2\psi}{dq^2}+3\frac{\partial\psi}{\partial q}+\left(e^{4q}-2e^{2q}\varepsilon_N+2E-\frac{3}{2}\right)\psi=0$$

The solution is

$$\psi_n(a)=A_n e^{-\frac{a^2}{2}}a^{\sqrt{2E+\frac{3}{4}}+\frac{3}{2}}F\left(-n,\sqrt{2E+\frac{3}{4}}+1,a^2\right)\qquad E_n=\frac{1}{2}\left(\varepsilon_N-2n-1\right)^2-\frac{3}{8}$$

Now we shall turn to **the non-conformal case** $\xi \neq 0$.

The action for the scalar field

$$S_{scal} = \int dt \int d^3x \sqrt{\gamma} \left[\frac{a}{N} \dot{\phi}^* \dot{\phi} + Na\phi^* \Delta\phi - \frac{N}{a} \phi^* \phi + \xi \left(-\frac{N}{a} \phi^* \phi + \frac{\dot{a}}{N} \dot{\phi}^* \phi + \frac{\dot{a}}{N} \phi^* \dot{\phi} - \frac{\dot{a}^2}{Na} \phi^* \phi \right) \right]$$

We consider the part of the action proportional to ξ as a perturbation. However, this part contains time derivatives of the scalar field as well as the scale factor.

In the perturbation theory, one can approximate the action using motion equations of the unperturbed problem and their solutions.

$$\dot{\phi}(\mathbf{x}, t) = i \frac{N}{a} \sum_{\mathbf{k}} \sqrt{\frac{\lambda_{\mathbf{k}}}{2}} \left[b_{\mathbf{k}}(t) \Phi_{\mathbf{k}}(\mathbf{x}) - c_{\mathbf{k}}^*(t) \Phi_{\mathbf{k}}^*(\mathbf{x}) \right]$$

For the scale factor, we should use the expansion $\quad \dot{a}(t) = \frac{z}{\varepsilon} + \frac{\varepsilon}{2} \ddot{a}(t') - \frac{\varepsilon^2}{3!} \dddot{a}(t') + \ldots \quad z = a(t') - a(t)$

It turns out that the term with $\dot{a}^2$ gives an additional contribution to the zeroth order in ε, so that in this order we have:

$$\chi = \hat{T} \chi$$

Consider the evolution operator

$$\hat{U}\left(\hat{b}^{\dagger}_{\mathbf{k}}, \hat{b}_{\mathbf{k}}, \hat{c}^{\dagger}_{\mathbf{k}}, \hat{c}_{\mathbf{k}}, \varepsilon\right) = \exp\left[-i\hat{H}\left(\hat{b}^{\dagger}_{\mathbf{k}}, \hat{b}_{\mathbf{k}}, \hat{c}^{\dagger}_{\mathbf{k}}, \hat{c}_{\mathbf{k}}\right)\varepsilon\right] = \exp\left[-i\left(\hat{H}_0 + \hat{V}\right)\varepsilon\right] =$$

$$= \exp\left(-i\hat{H}_0\varepsilon\right)\exp\left(-i\hat{V}\varepsilon\right)\exp\left(\frac{1}{2}\varepsilon^2\left[\hat{H}_0, \hat{V}\right]\right) = \exp\left(-i\hat{H}_0\varepsilon\right) - i\varepsilon\hat{V}\exp\left(-i\hat{H}_0\varepsilon\right) + \ldots$$

We know that the unperturbed Hamiltonian $\hat{H}_0$ does not change a Fock basis, while the operator $\left(\hat{H}_0 + \hat{V}\right)$ causes the basis to alter every time instant.

One can say that the evolution operator $\hat{U} = \exp\left[-i\left(\hat{H}_0 + \hat{V}\right)\varepsilon\right]$ maps the basis at some initial time moment $\left|b^{(0)}_{\mathbf{k}}, c^{(0)}_{\mathbf{k}}\right\rangle$ to the basis at the subsequent time moment $|b'_{\mathbf{k}}, c_{\mathbf{k}}{}'\rangle$. But, when we deal with the perturbation theory, we should use the original basis $\left|b^{(0)}_{\mathbf{k}}, c^{(0)}_{\mathbf{k}}\right\rangle$.

The wave function is a projection of the state vector onto some basis, so, it contains information about the state vector as well as about the basis.

While Feynman used the equation for the wave function, it would be preferrable for us to use an appropriate formula for the state vector.

$$\psi\left(x(t+\varepsilon),t+\varepsilon\right)=\int\exp\left[\frac{i}{\hbar}S\left(x(t+\varepsilon),x(t)\right)\right]\psi\left(x(t),t\right)\frac{dx(t)}{A}$$

$$\left|\Psi(t+\varepsilon)\right\rangle=\hat{U}(t,t+\varepsilon)\left|\Psi(t)\right\rangle=$$

$$=\int\hat{U}(t,t+\varepsilon)\left|b_{\mathbf{k}}^{(0)},c_{\mathbf{k}}^{(0)}\right\rangle\left\langle b_{\mathbf{k}}^{(0)},c_{\mathbf{k}}^{(0)}\middle|\Psi(t)\right\rangle\prod_{\mathbf{k}}\exp\left(-b_{\mathbf{k}}^{*}b_{\mathbf{k}}-c_{\mathbf{k}}^{*}c_{\mathbf{k}}\right)\frac{db_{\mathbf{k}}^{*}db_{\mathbf{k}}}{2\pi i}\frac{dc_{\mathbf{k}}^{*}dc_{\mathbf{k}}}{2\pi i}$$

$$\left\langle b_{\mathbf{k}}^{(0)},c_{\mathbf{k}}^{(0)}\middle|\Psi(t+\varepsilon)\right\rangle=$$

$$=\int\left\langle b_{\mathbf{k}}^{(0)},c_{\mathbf{k}}^{(0)}\right|\exp\left(-i\hat{H}_0\varepsilon\right)\left|b_{\mathbf{k}'}^{(0)},c_{\mathbf{k}'}^{(0)}\right\rangle\left\langle b_{\mathbf{k}'}^{(0)},c_{\mathbf{k}'}^{(0)}\middle|\Psi(t)\right\rangle\prod_{\mathbf{k}'}\exp\left(-b_{\mathbf{k}'}^{*}b_{\mathbf{k}'}-c_{\mathbf{k}'}^{*}c_{\mathbf{k}'}\right)\frac{db_{\mathbf{k}'}^{*}db_{\mathbf{k}'}}{2\pi i}\frac{dc_{\mathbf{k}'}^{*}dc_{\mathbf{k}'}}{2\pi i}+$$

$$+i\varepsilon\int\left\langle b_{\mathbf{k}}^{(0)},c_{\mathbf{k}}^{(0)}\right|\hat{V}\exp\left(-i\hat{H}_0\varepsilon\right)\left|b_{\mathbf{k}'}^{(0)},c_{\mathbf{k}'}^{(0)}\right\rangle\left\langle b_{\mathbf{k}'}^{(0)},c_{\mathbf{k}'}^{(0)}\middle|\Psi(t)\right\rangle\prod_{\mathbf{k}'}\exp\left(-b_{\mathbf{k}'}^{*}b_{\mathbf{k}'}-c_{\mathbf{k}'}^{*}c_{\mathbf{k}'}\right)\frac{db_{\mathbf{k}'}^{*}db_{\mathbf{k}'}}{2\pi i}\frac{dc_{\mathbf{k}'}^{*}dc_{\mathbf{k}'}}{2\pi i}$$

In the right-hand side, we have obtained a perturbation theory expansion.

Let us reorganize the left-hand side.

$$\left|\Psi(t+\varepsilon)\right\rangle = \left|\Psi(t)\right\rangle + \varepsilon\frac{d}{dt}\left|\Psi(t)\right\rangle + \ldots$$

$$\left\langle b_{\mathbf{k}}^{(0)}, c_{\mathbf{k}}^{(0)}\middle|\Psi(t+\varepsilon)\right\rangle = \left\langle b_{\mathbf{k}}^{(0)}, c_{\mathbf{k}}^{(0)}\middle|\Psi(t)\right\rangle + \varepsilon\left\langle b_{\mathbf{k}}^{(0)}, c_{\mathbf{k}}^{(0)}\middle|\frac{d}{dt}\middle|\Psi(t)\right\rangle + \ldots =$$

$$= \Psi\left(b_{\mathbf{k}}^{*}, c_{\mathbf{k}}^{*}, t\right) + \varepsilon\frac{d}{dt}\left\langle b_{\mathbf{k}}^{(0)}, c_{\mathbf{k}}^{(0)}\middle|\Psi(t)\right\rangle - \varepsilon\left(\frac{d}{dt}\left\langle b_{\mathbf{k}}^{(0)}, c_{\mathbf{k}}^{(0)}\right|\right)\left|\Psi(t)\right\rangle + \ldots$$

On the other hand, we have another basis $|b_{\mathbf{k}}', c_{\mathbf{k}}'\rangle$ at the moment $(t+\varepsilon)$,

$$\left|b_{\mathbf{k}}', c_{\mathbf{k}}'\right\rangle = \hat{P}\left|b_{\mathbf{k}}^{(0)}, c_{\mathbf{k}}^{(0)}\right\rangle = \left(\hat{1}+\hat{T}\right)\left|b_{\mathbf{k}}^{(0)}, c_{\mathbf{k}}^{(0)}\right\rangle = \left|b_{\mathbf{k}}^{(0)}, c_{\mathbf{k}}^{(0)}\right\rangle + \hat{T}\left|b_{\mathbf{k}}^{(0)}, c_{\mathbf{k}}^{(0)}\right\rangle$$

$$\hat{T}\left|b_{\mathbf{k}}, c_{\mathbf{k}}, t\right\rangle = \varepsilon\frac{d}{dt}\left|b_{\mathbf{k}}, c_{\mathbf{k}}, t\right\rangle = \varepsilon\lim_{\varepsilon\to 0}\frac{1}{\varepsilon}\left(\left|b_{\mathbf{k}}, c_{\mathbf{k}}, t+\varepsilon\right\rangle - \left|b_{\mathbf{k}}, c_{\mathbf{k}}, t\right\rangle\right) = \left|b_{\mathbf{k}}, c_{\mathbf{k}}, t+\varepsilon\right\rangle - \left|b_{\mathbf{k}}, c_{\mathbf{k}}, t\right\rangle = \left|b_{\mathbf{k}}', c_{\mathbf{k}}'\right\rangle - \left|b_{\mathbf{k}}^{(0)}, c_{\mathbf{k}}^{(0)}\right\rangle$$

$$\left\langle b_{\mathbf{k}}^{(0)}, c_{\mathbf{k}}^{(0)}\middle|\Psi(t+\varepsilon)\right\rangle = \Psi\left(b_{\mathbf{k}}^{*}, c_{\mathbf{k}}^{*}, t\right) + \varepsilon\frac{d}{dt}\Psi\left(b_{\mathbf{k}}^{*}, c_{\mathbf{k}}^{*}, t\right) - \left\langle b_{\mathbf{k}}^{(0)}, c_{\mathbf{k}}^{(0)}\middle|\hat{T}\middle|\Psi(t)\right\rangle =$$

$$= \Psi\left(b_{\mathbf{k}}^{*}, c_{\mathbf{k}}^{*}, t\right) - \hat{T}\Psi\left(b_{\mathbf{k}}^{*}, c_{\mathbf{k}}^{*}, t\right) + \varepsilon\frac{d}{dt}\Psi\left(b_{\mathbf{k}}^{*}, c_{\mathbf{k}}^{*}, t\right)$$

The explicit form of the operator $\hat{T}$ is obtained when deriving the Schrödinger equation.

$$\hat{T}\chi = \sum_{\mathbf{k}}\left[-\frac{\xi}{a^2\lambda_{\mathbf{k}}}\left(b_{\mathbf{k}}^* c_{\mathbf{k}}^* + b_{\mathbf{k}}^*\frac{\partial}{\partial b_{\mathbf{k}}^*} + c_{\mathbf{k}}^*\frac{\partial}{\partial c_{\mathbf{k}}^*} + \frac{\partial}{\partial b_{\mathbf{k}}^*}\frac{\partial}{\partial c_{\mathbf{k}}^*}\right)\right]\chi$$

The Schrödinger equation for gravitational and non-conformal scalar fields (without ghosts):

$$\begin{aligned}
i\frac{\partial\Psi}{\partial t} = &-\frac{1}{2M}\frac{\partial}{\partial Q^{\mu}}\left(MG^{\mu\nu}\frac{\partial\Psi}{\partial Q^{\nu}}\right) - \frac{1}{2}Na\Psi + \frac{N}{a}\sum_{\mathbf{k}}\lambda_{\mathbf{k}}\left(b_{\mathbf{k}}^*\frac{\partial\Psi}{\partial b_{\mathbf{k}}^*} + c_{\mathbf{k}}^*\frac{\partial\Psi}{\partial c_{\mathbf{k}}^*}\right) + \\
&+\xi\left[\frac{N}{a^2}\left(\frac{\partial}{\partial a} + \frac{df}{da}\frac{\partial}{\partial N}\right)\sum_{\mathbf{k}}\left(b_{\mathbf{k}}^* c_{-\mathbf{k}}^*\Psi - \frac{\partial\Psi}{\partial b_{\mathbf{k}}^*}\frac{\partial\Psi}{\partial c_{\mathbf{k}}^*}\right) - \right. \\
&\left. -\sum_{\mathbf{k}}\left(\frac{1}{\lambda_{\mathbf{k}}a^2}\left[-\frac{1}{2M}\frac{\partial}{\partial Q^{\mu}}\left(MG^{\mu\nu}\frac{\partial}{\partial Q^{\nu}}\right) - \frac{1}{2}Na\right]\left[b_{\mathbf{k}}^*\frac{\partial\Psi}{\partial b_{\mathbf{k}}^*} + c_{\mathbf{k}}^*\frac{\partial\Psi}{\partial c_{\mathbf{k}}^*} + b_{\mathbf{k}}^* c_{-\mathbf{k}}^*\Psi + \frac{\partial\Psi}{\partial b_{\mathbf{k}}^*}\frac{\partial\Psi}{\partial c_{\mathbf{k}}^*}\right]\right)\right]
\end{aligned}$$

$$G^{\mu\nu} = \begin{pmatrix} \left(\frac{df}{da}\right)^2 & \frac{df}{da} \\ \frac{df}{da} & -\frac{N}{a} \end{pmatrix}$$

Directions of the future research:

- To calculate probabilities of transitions between vacuum state (no particles) and states containing pairs of particles.
- To evaluate probability that created virtual pairs of particles would become real particles.
- To assess probability of the scenarios where the Universe was created in a state with a large value of the parameter *E*, or most created particles annihilate. Transitions to states with a smaller value of *E* are probable.
- To compare our results with those obtained in the canonical framework using diagonalization of the Hamiltonian. In particular, it concerns a relation between Fock bases in different moments of time.